Properties of Multiplication

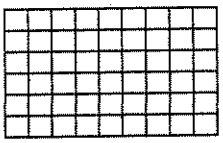
Mathematical Focus

- Identify and use the commutative property of multiplication, and the distributive property of multiplication over addition.
- Identify and use one as the identity element of multiplication.
- Identify and use the zero principle in multiplication.

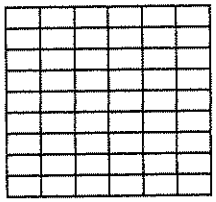
Potential Challenges and Misconceptions

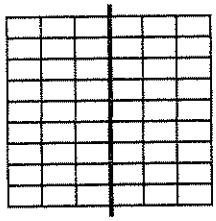
Properties of arithmetic provide the conceptual underpinnings for computational strategies and the foundation for algebraic thinking. It is important to introduce these properties at the basic fact level so that students can apply them easily as multiplication examples become more complex.

The array or area model of multiplication is particularly powerful for modeling the commutative and distributive properties of multiplication. By turning the rectangle or looking at it sideways, students can see that the total stays the same, that $a \times b = b \times a$. The distributive property of multiplication over addition states that $a(b + c) = ab + ac$. It is this property that allows us to find 9×6 or $9 \times (3 + 3)$ by realizing that this is the same as $(9 \times 3) + (9 \times 3)$ or $2(9 \times 3)$. (See figure.) So if students know the product when multiplying by three, they can double it to multiply by six.



$6 \times 9 = 9 \times 6$
commutative property of multiplication





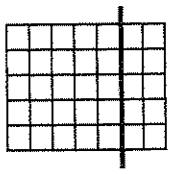
$9(3 + 3)$
$= 9 \times 3 + 9 \times 3$
distributive property of multiplication over addition

Though not helpful to the learning of basic facts, multiplication is also associative, that is, $(a \times b) \times c = a \times (b \times c)$. This property is useful when multiplying three numbers, for example, $7 \times 2 \times 5$. We can choose to start with 2×5, rather than 7×2.

In the Classroom

One teacher arranges students in groups and gives each group some tiles and directions such as *Build 3 rows of 4*. For each direction she gives the reversed order to another group, in this case, *Build 4 rows of 3*. (She also avoids giving another group *Build 2 rows of 6*, as it would yield the same total.) When the arrays are built, the groups leave their building directions by their arrays, walk around to find another group who used the same number of tiles as they did, and record that group's directions and a sketch of the array. Once students have made these recordings, they return to their seats, compare their own directions and array with what they recorded, and write about their observations. One student summarizes his thinking by saying, "You can just turn these around so you have to get the same number."

On another day, the teacher shows a 5-by-7 array and has students mark the same area on graph paper. Once the students agree that the array represents $5 \times 7 = 35$, the teacher invites a student to come up and use a marker to thicken any line in the array (see figure).



$5 \times 5 = 25$
$5 \times 2 = 10$
$5 \times 7 = 35$

Students mark their arrays this way as well and talk about what they see. The class repeats the activity until students seem comfortable that any array can be subdivided and totals of the smaller rectangles can be added to find the total product. The teacher identifies this as the distributive property of multiplication over addition. Then the teacher assigns students to work in pairs to complete the *Toothpick Distributive Property* reproducible (see page A12 in the appendix). Together students place a toothpick on various horizontal and vertical line segments until they find an arrangement they prefer. Then they record multiplication sentences represented by the two rectangles and use them to find the product modeled by the original array.

Meeting Individual Needs

Knowing the identity element, the zero principle, and the commutative property eliminates sixty-four of the facts from 0×0 to 9×9. Anxious students may be overwhelmed by the notion of learning one hundred basic multiplication facts. Give each student a multiplication table of all one hundred facts and have them cross out the sixty-four they already know.

REFERENCES/FURTHER READING

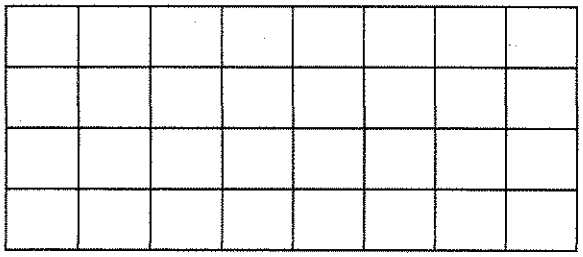
Tent, Margaret. 2006. "Understanding the Properties of Arithmetic: A Prerequisite for Success in Algebra." *Mathematics Teaching in the Middle School* 12 (1): 22–25.

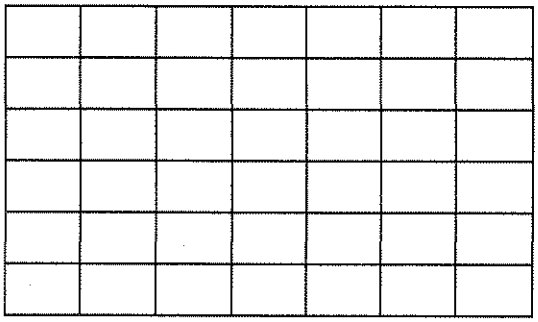
Wallace, Ann, and Susan Gurganus. 2005. "Teaching for Mastery of Multiplication." *Teaching Children Mathematics* 12 (1): 26–33.

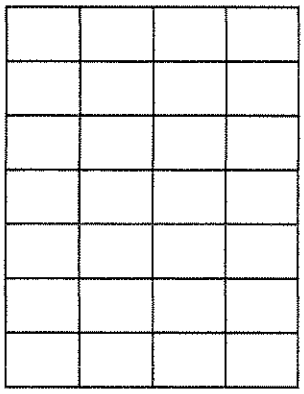
TOOTHPICK DISTRIBUTIVE PROPERTY

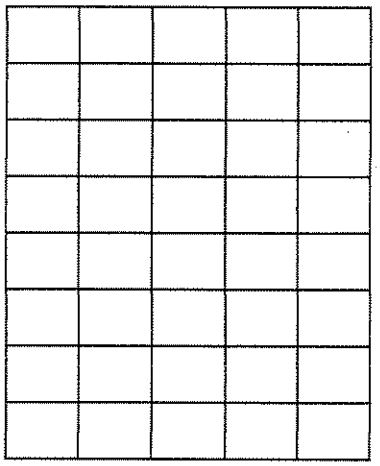
Name: Date:

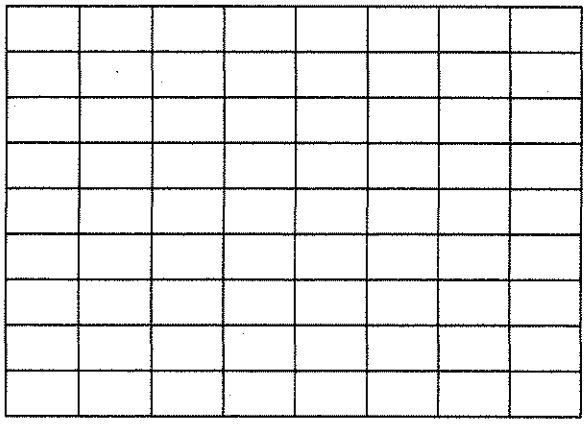
For each array, place a toothpick on one of the line segments to make two rectangles. Try it in different places until you find the one you like best. Write two multiplication sentences for the two rectangles you made. Then lift your toothpick and write the multiplication sentence for the whole array.

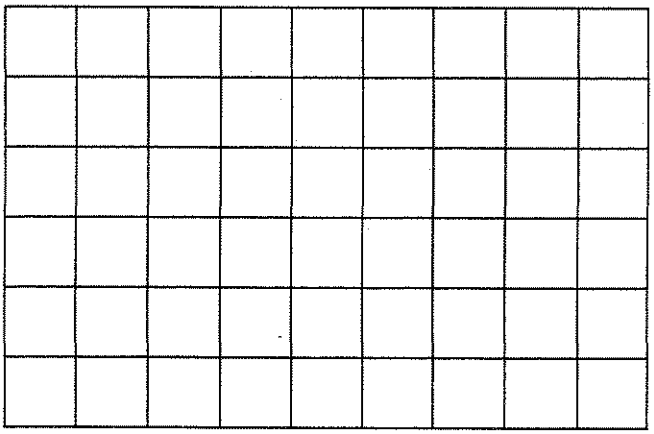
1. 

2. 

3. 

4. 

5. 

6. 

7. What do you notice about your answers?